Intuitive “proofs”

Competitive Programming: Core Skills

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SPbSU

Introduction

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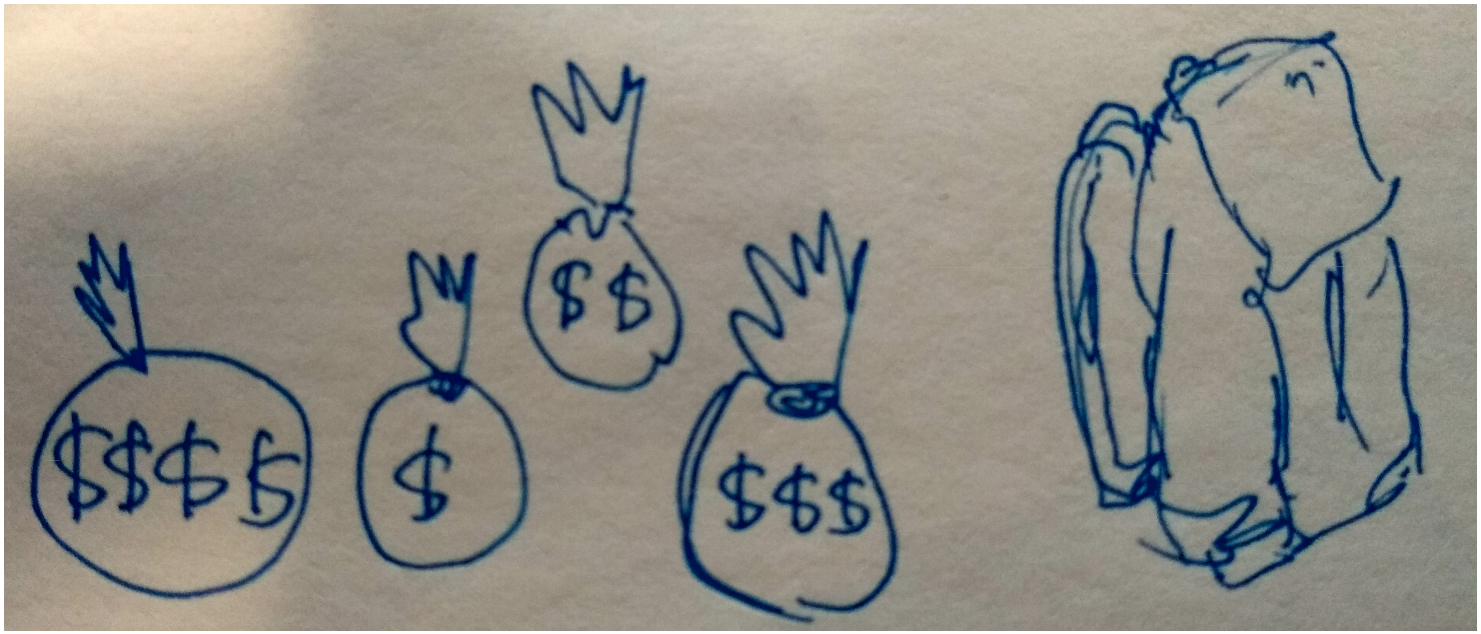
- Often you manage to convince yourself that it is without any hard proof.
- That can play a trick.

Robber's problem



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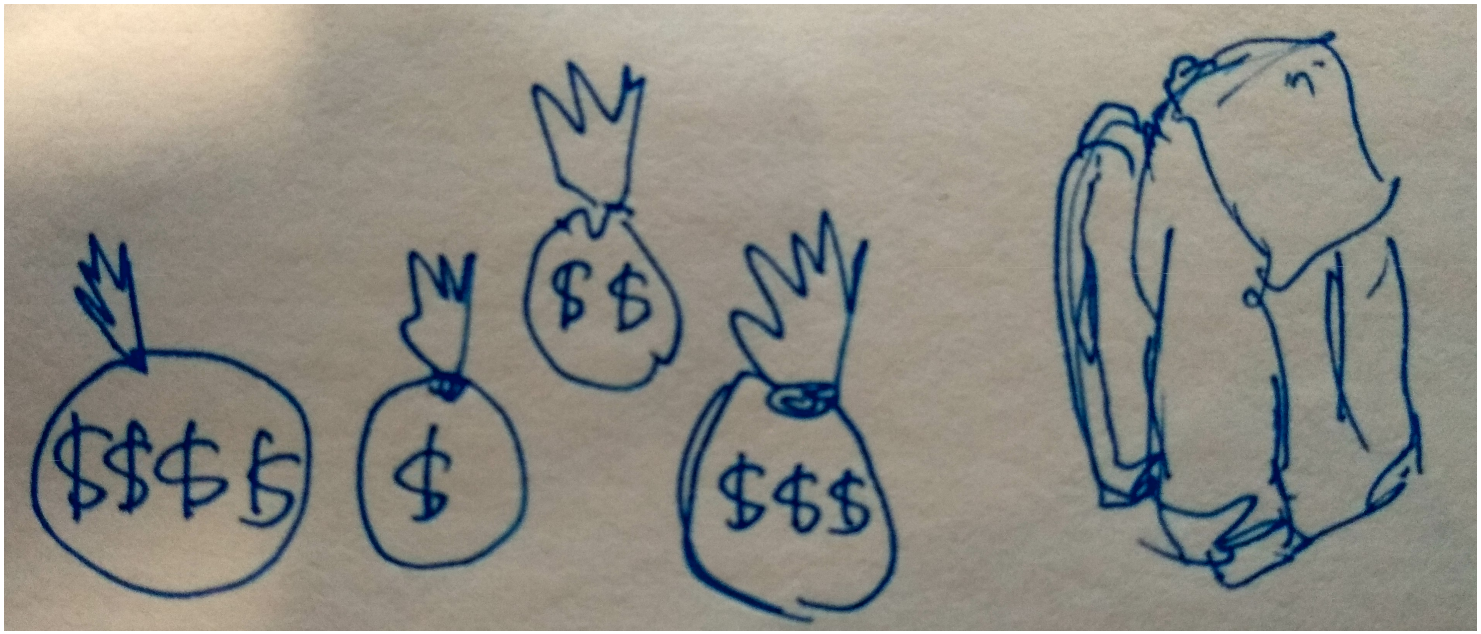

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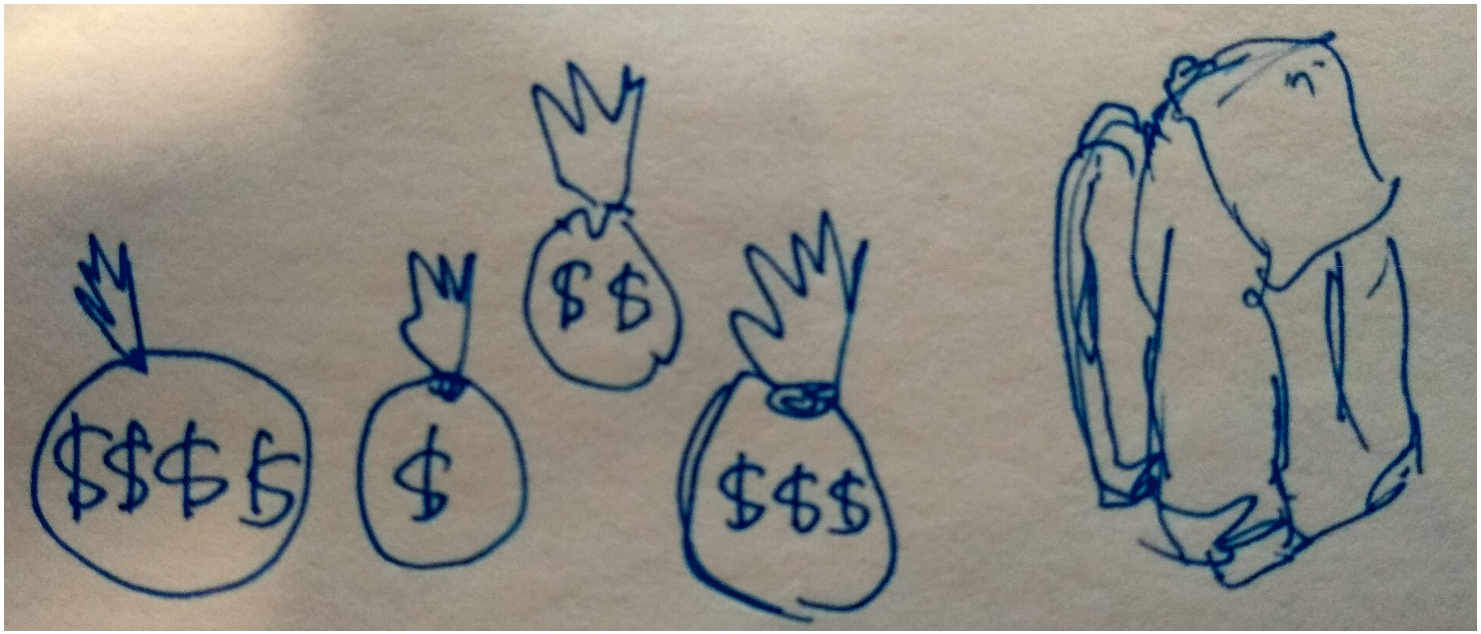

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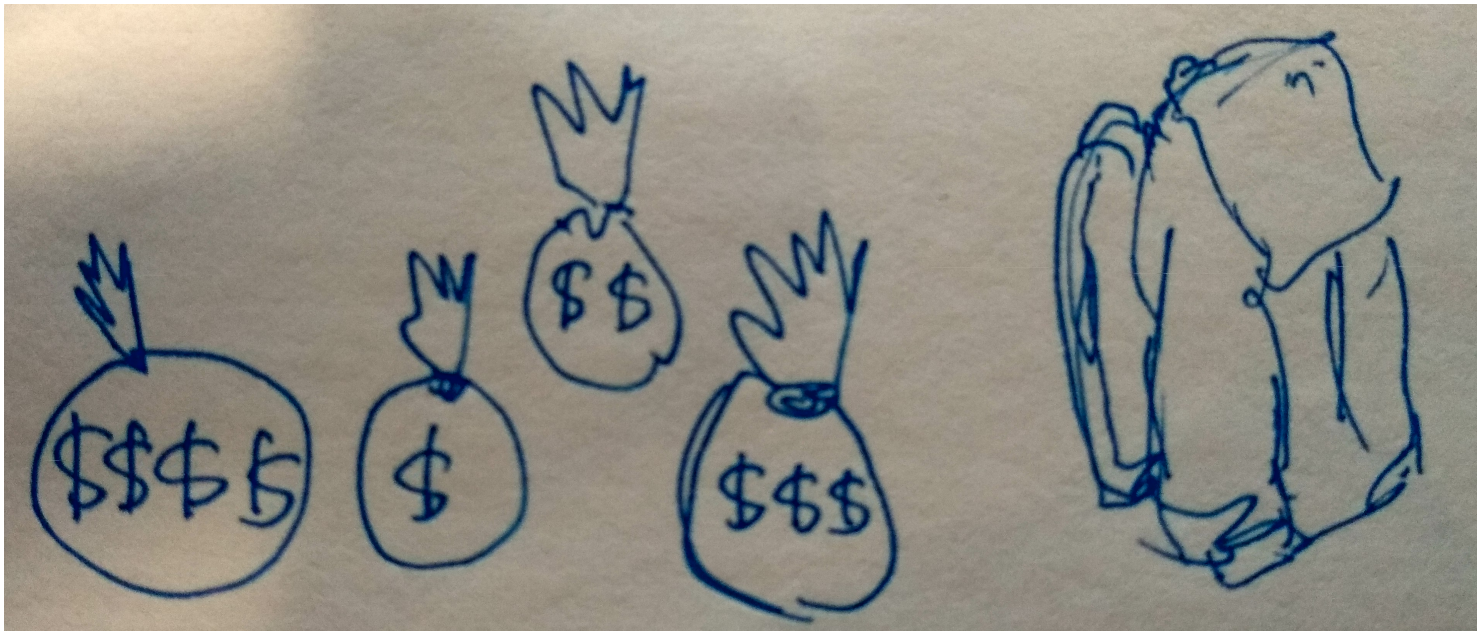

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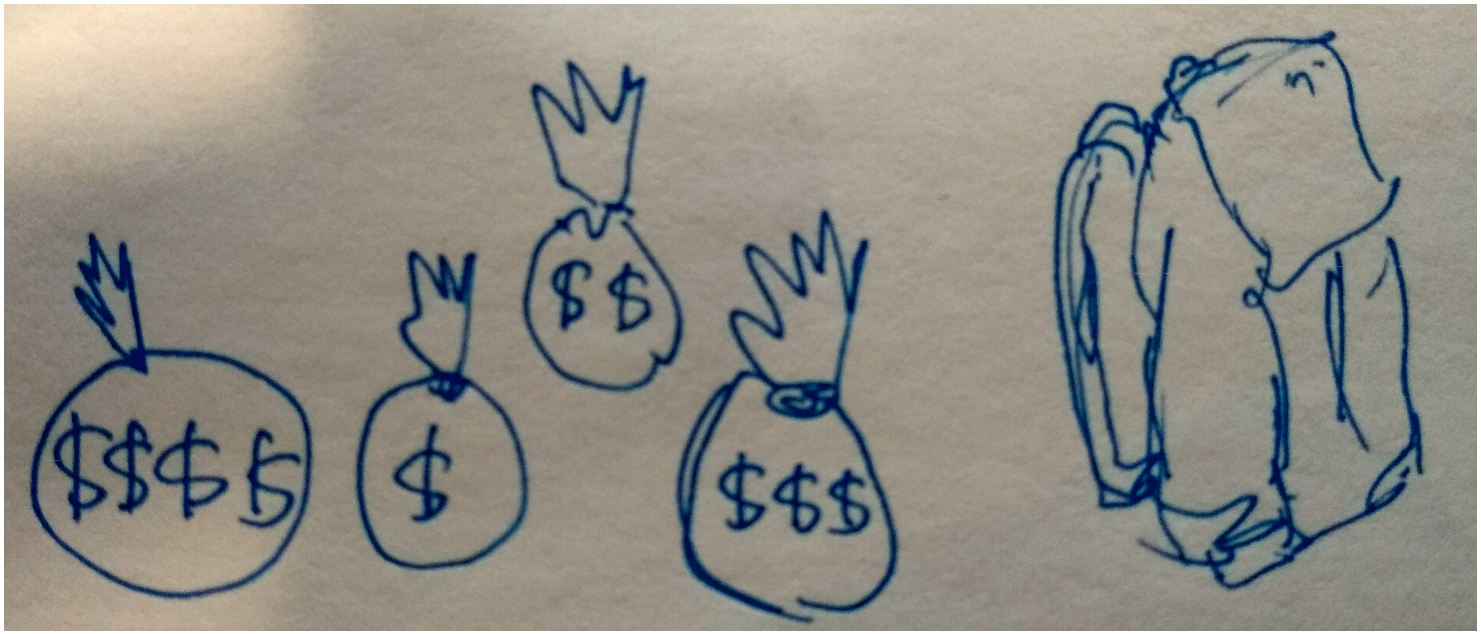

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- There are n valuable items with volumes $v_1, \ldots, v_n$ (kg) and costs $c_1, \ldots, c_n$ ($)
- You can put items of total volume at most V.
- What is the largest total cost of items you can steal?

Tempting Approach

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- We should maximize cost per volume unit value.
- So let's calculate utility $\frac{c_i}{v_i}$ for each item.
- The better the utility the better the item.
- Therefore we should try to put items with maximal utility first.
- Nice and easy. But, unfortunatelly, wrong.

Example

$v_1 = 3$

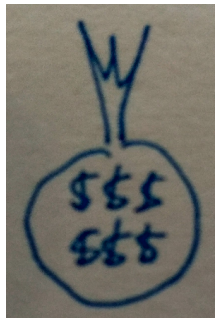
$c_1 = 2$

$\frac{2}{3}$

$v_2 = 2$

$c_2 = 3$

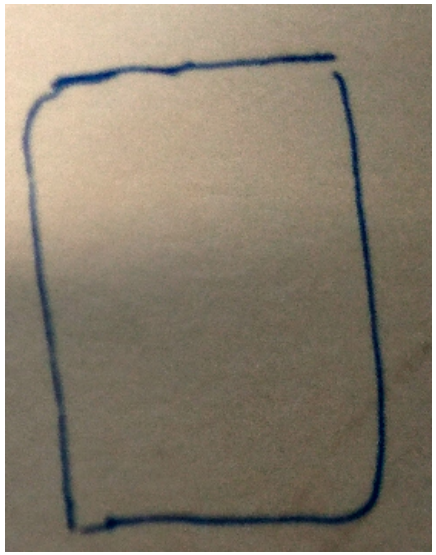
$\frac{3}{2}$



$v_3 = 5$

$c_3 = 6$

$\frac{6}{5}$



$V = 5$

Example

The Best

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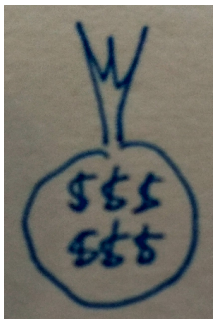

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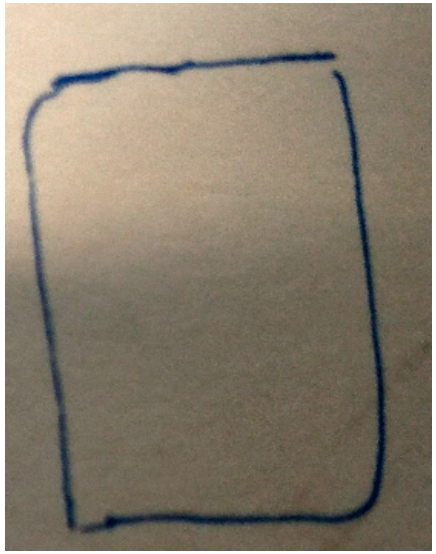

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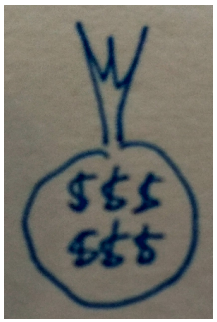

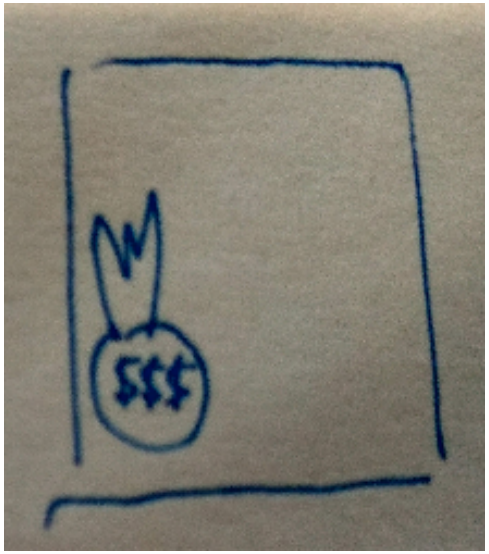

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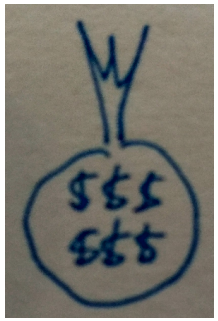

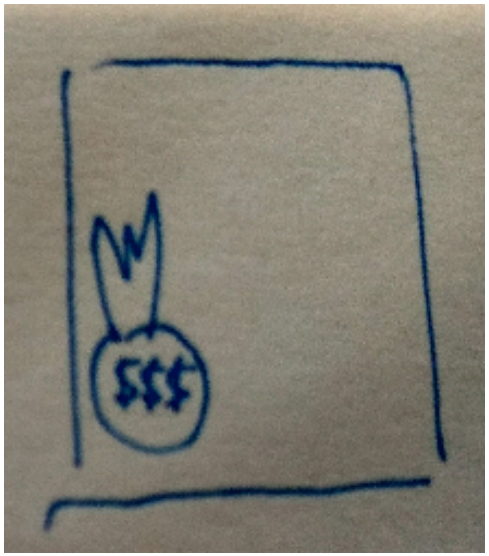

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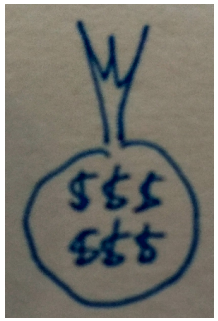

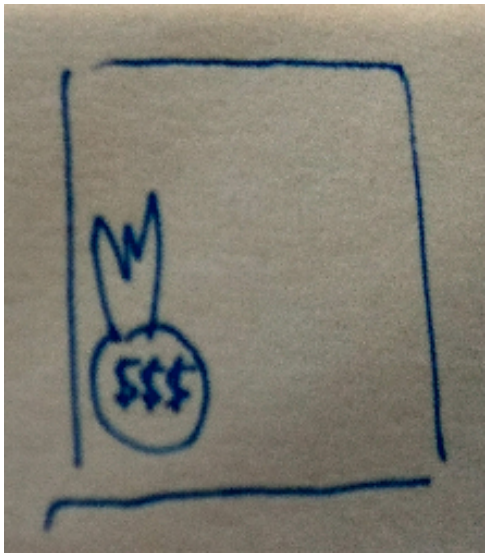

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But the third item doesn't fit to the knapsack.

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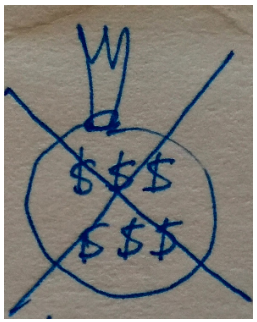

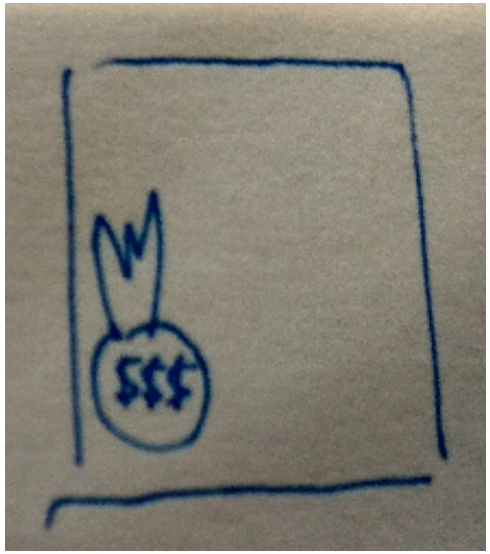

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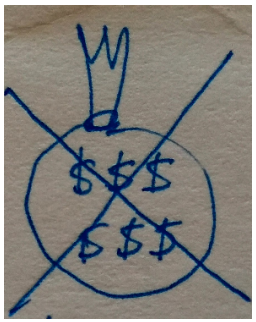

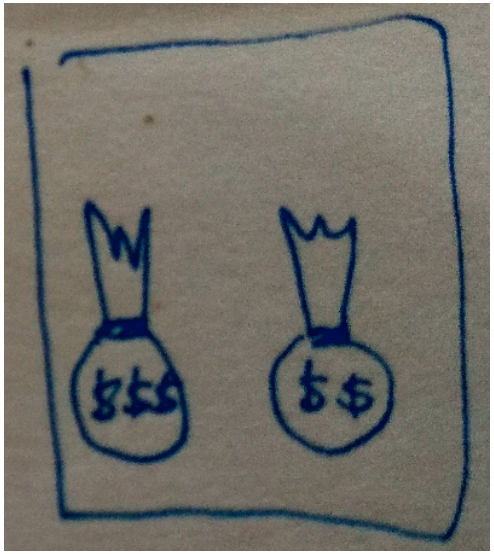

$v_1 = 3$ $\quad$ $v_3 = 5$

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$V = 0;\ C = 5$

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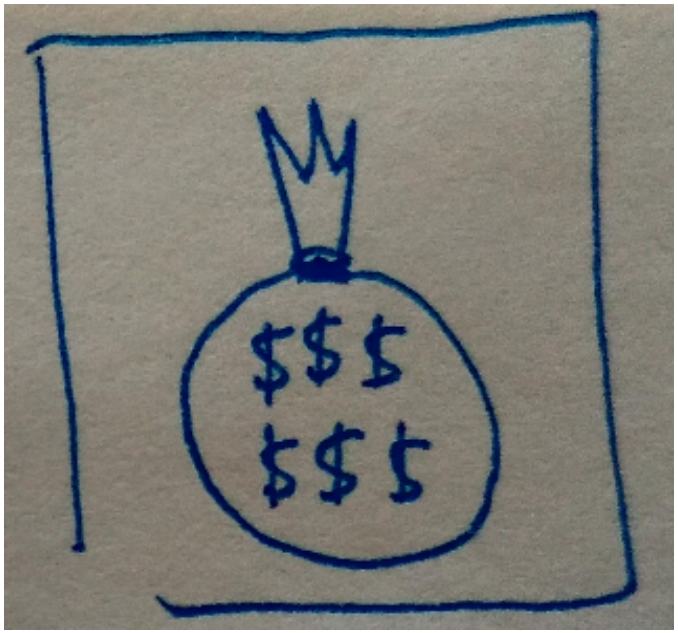
- But we could do better with the third item only:



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- Then how to check if approach is wrong?
- The simplest thing to do is to check your algorithm with pen and paper against sample tests.
- But what to do if your solution got wrong answer 47 and you have no idea what to do?
- It'd be good to have some basic solution which is always conceptually correct.
- And that's what we'll do!