Brute force solutions

Competitive Programming: Core Skills

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SPbSU

Outline

Introduction

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- But sometimes it's not.
- In this lesson we are going to develop a method for designing solutions which are always correct.
- The catch is they are going to be slow.

Digits ordering

Largest number

Input: list of digits.

Output: the largest number that can be made of the digits.

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Sample output: 735.

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Largest number

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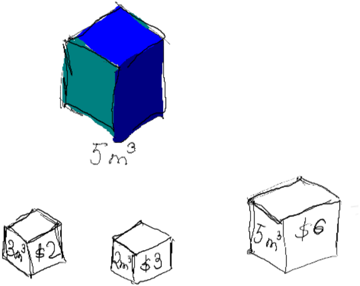
Output: the largest number that can be made of the digits.

Sample input: 3,7,5
Sample output: 735.
The solution is to order the digits from the biggest one to the smallest one.

Robber's problem (aka knapsack problem)



Robber's problem

Input: a list of items with weights (kg) and costs ($) as well as the capacity of a bag (kg).
Output: the maximum total cost of items that fit in the bag.

Tempting Approach

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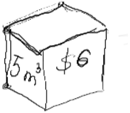
- Let's calculate utility $\frac{\text{cost}}{\text{weight}}$ for each item.
- The better the utility the better the item.
- Therefore we should try to put items with maximum utility first.
- Nice and easy. But, unfortunately, wrong.

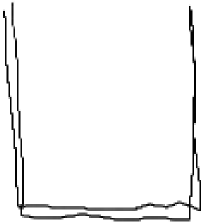
Example

3 kg
$2
$\frac{2}{3}$

2 kg
$3
$\frac{3}{2}$



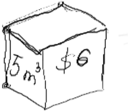
5 kg
$6
$\frac{6}{5}$

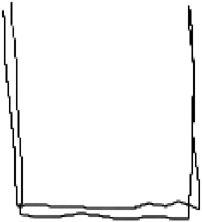


$V = 5$

Example

The Best





3 kg	2 kg	5 kg
\$2	\$3	\$6
$\frac{2}{3}$	$\frac{3}{2}$	$\frac{6}{5}$

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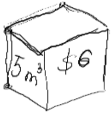

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\$6

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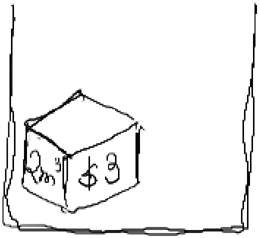
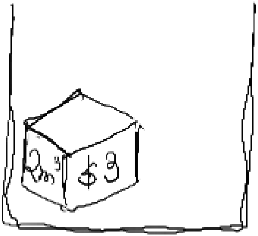


$V = 3;\ C = 3$

Example



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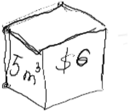

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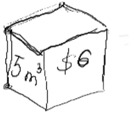

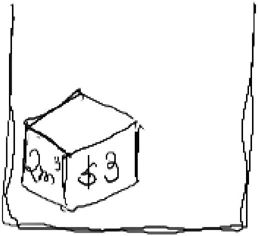

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$V = 3; \, C = 3$

But the third item doesn't fit to the knapsack.

Example

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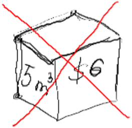

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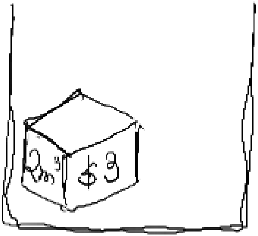

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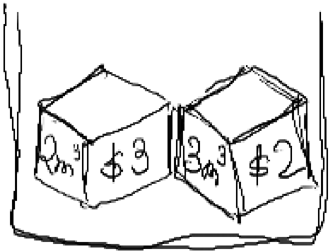

$V = 0;\ C = 5$

Example

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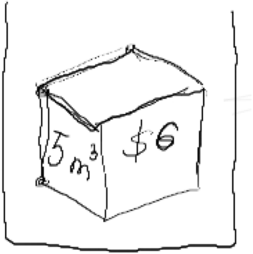
- But we could do better with the third item only:



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- It'd be good to have a solution which is always conceptually correct.
- And that's what we'll do!

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- The approach yields slow solutions but it's conceptually correct by definition.
- Therefore it could be used to verify correctness of faster solutions for the same problem.

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We will call the set A search space.

Examples

Superstring

Given m strings $s_1, \ldots, s_m$ consisting of letters "a" and "b" only and an integer n. Find a string s of length n containing each s_i (for all $1 \leq i \leq m$) as a substring.

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Sample

Input: $m = 2$; $n = 3$; $s_1 =$ ab, $s_2 =$ ba

Output: aba (aba, aba) (another valid output is bab)

Search space

- One way to solve a problem is to simply go through all possible candidate solutions. For the superstring problem, the search space consists of all strings of length n over the alphabet $\{a, b\}$. For each such string, we check whether it is indeed a superstring of $s_1, \ldots, s_m$

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- Let's consider another testcase: $n = 4$, $s_1 = \texttt{bab}$, $s_2 = \texttt{abb}$.
- There are only $2^4 = 16$ strings of four letters "a" and "b".

Search space

Candidate	bab	abb	Candidate	bab	abb
aaaa	×	×	baaa	×	×
aaab	×	×	baab	×	×
aaba	×	×	baba	baba	×
aabb	×	aabb	**babb**	babb	babb
abaa	×	×	bbaa	×	×
abab	abab	×	bbab	bbab	×
abba	×	×	abba	×	×
abbb	×	abbb	bbbb	×	×

Examples

Maximum subarray problem

Given an array $a_1, \ldots, a_n$. What is the largest possible sum $a_l + a_{l+1} + \ldots + a_{r-1} + a_r$ for $1 \le l \le r \le n$?
Note that a_i could be negative.

Sample

...

Input: $a = (4, 1, -2, 3, -10, 5)$

Output: The best subarray is $(\underbrace{4, 1, -2, 3}, -10, 5)$ and the sum is $4 + 1 + (-2) + 3 = 6$.

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- Enumerate all pairs (l, r) such that $l \leq r$, for each pair compute the sum $a_l + \cdots + a_r$, and take the maximum.

Examples

Robber's problem

you have a knapsack of volume W and n items of volumes $w_1, \ldots, w_n$ and costs $c_1, \ldots, c_n$. What is the largest total cost of the set of items whose total weight does not exceed W?

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Input: $V = 5$; $n = 3$

$v_1 = 3 \quad v_2 = 2 \quad v_3 = 5$

$c_1 = 2 \quad c_2 = 3 \quad c_3 = 6$

Output: The best solution is to put the last item to the knapsack and get the total cost 6.

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Search space: summary

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- For the maximum subarray problem the search space gives us the solution instantly.
- We can try all possible pairs with two nested `for` cycles.
- For the substring problem we want to try all possible strings of n symbols.
- It'd be good to have n nested `for` cycles iterating through letters “a” and “b”.

n nested for cycles

Your pseudo-Python code will look like this for the superstring problem:

```
for x[0] in ['a','b']:
    for x[1] in ['a','b']:
        # ...
        for x[n-1] in ['a','b']:
            # check if x contains
            # all strings s[1], ... s[m]
```

Outline

1. Intuitive solutions
2. Search space
3. Backtracking

Introduction

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- Backtracking is roughly the way how to write n nested `for` cycles.

Enumerating all strings x over $\{a, b\}$ of length n:

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for x[0] in ['a','b']:
    for x[1] in ['a','b']:
        # ...
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            # do something with x
```

Recursion

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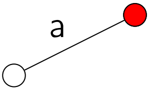
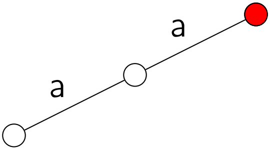
The simplest possible way to simulate this “code” with an actual code is via recursion.


a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The key idea is to look at n nested `for` cycles like this:

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So we can implement the function recursively.

Resursion

We will write the function `nestedFors` with additional parameter `firstFor` and it'll behave like

firstFor	Behaviour
0	`threeFors`
1	`twoFors`
2	`oneFor`
3	`print(x)`

```
def nestedFors(n, firstFor, x):
    if firstFor < n:
        for x[firstFor] in ['a','b']:
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    else:
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Analyse the code

When we do have another `for` cycles to run:

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we call our function recursively, starting with the next `for`.

Analyse the code

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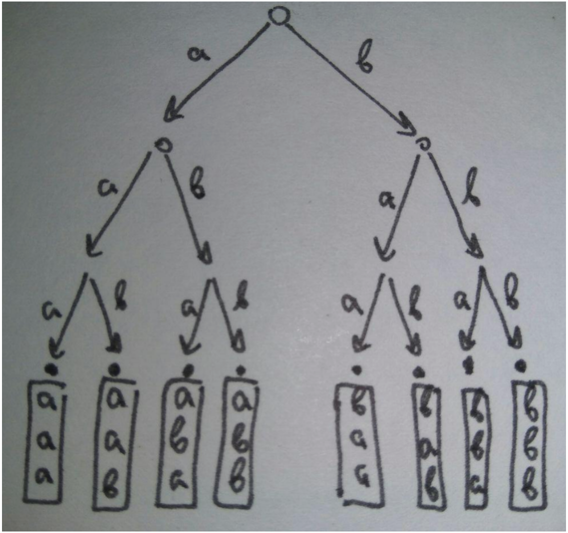
```

we call our function recursively, starting with the next `for`. Or we are in the deepest level of the recursion and we just do the job with x:

```
print(x)
```

Visualisation



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- How to enumerate all sets of n items?
- Basically, it is the same as enumerating all strings over $\{0, 1\}$ of length n!

Set to string

Set	Items		
	1	2	3
$\varnothing$	0	0	0
$\{1\}$	1	0	0
$\{2\}$	0	1	0
$\{1,2\}$	1	1	0
$\{3\}$	0	0	1
$\{1,3\}$	1	0	1
$\{2,3\}$	0	1	1
$\{1,2,3\}$	1	1	1

Robber's problem

Recall our example: $n = 3$; $V = 5$ and

$v_1 = 3 \quad v_2 = 2 \quad v_3 = 5$

$c_1 = 2 \quad c_2 = 3 \quad c_3 = 6$

Items 1	2	3	Set	Weight	Cost
0	0	0	$\varnothing$	0	0
1	0	0	$\{1\}$	3	2
0	1	0	$\{2\}$	2	3
1	1	0	$\{1,2\}$	$2+3=5$	$3+2=5$
0	0	1	$\{3\}$	5	6
1	0	1	$\{1,3\}$	$3+5=8$	$2+6=8$
0	1	1	$\{2,3\}$	$2+5=7$	$3+6=9$
1	1	1	$\{1,2,3\}$	$2+3+5=10$	$3+2+6=11$

Robber's problem

Therefore we reduced robber's problem to the same n nested for cycles!

Conclusion

Designing a brute force solution:

1. Identify the search space
2. Design a way of enumerating all its elements
3. Turn it into a solution

The resulting solution is usually slow, but: it is clearly correct and can be used for debugging